REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the revised indexing system printed in Volume 28, Number 128, October 1974, pages 1191–1194.

1 [4, 5].—G. I. Marchuk, *Methods of Numerical Mathematics,* Springer-Verlag, New York, Heidelberg, Berlin, 1975, viii + 316 pp., 24cm. Price $29.80.

This book is a translation of the original Russian edition which was published in 1973. The title is quite general, but the main content is numerical methods for initial or boundary value problems in Mathematical Physics.

After an opening chapter with fundamentals about difference schemes, a chapter follows dealing with Ritz and Galerkin's methods. The methods are illustrated on simple model equations, and various ways of choosing the subspaces for the finite element method are discussed.

Chapter 3 contains a discussion of the most commonly used methods for solving linear systems of algebraic equations. There is a general description of the most interesting iterative methods, such as SOR, Chebyshev acceleration, conjugate gradients and various splitting methods. The Fast Fourier Transform with different applications is also discussed, but there is nothing about the realization of general direct methods commonly used for finite element problems.

Chapter 4 deals with implicit methods for nonstationary problems, and there is a thorough discussion of splitting methods. The so-called component by component splitting methods, where only one component of the space operator is involved in each substep, receive special attention. They are considered by the author as the most important methods for applications.

Inverse problems, often ill posed, are usually not treated in books of this type, but regarding the importance of such problems, the inclusion of Chapter 5 is a very good idea. Here the problem of finding the coefficients of an operator or the initial state, given the current state, is discussed. Fourier series methods and methods based on perturbation theory are presented.

The discretization of the simplest problems of mathematical physics is discussed in Chapter 6. The Poisson, heat and wave equations are treated as well as the equation of motion, all in the simplest linear form. Only second order schemes are considered, but there is a section about increasing the accuracy by Richardson extrapolation.

Chapter 7 is devoted to the transport equation of radiative transfer theory. Different geometries are discussed, and the splitting technique is again applied.

Chapter 8 is a review of methods in numerical mathematics. The content is based on a talk given by the author at the International Congress of Mathematicians in Nice 1970.

As the author says in the preface, he has concentrated on basic ideas, and that is a good approach. The methods are presented in such a way that they can be easily generalized to more complicated problems. However, since the author emphasizes implicit methods, it would have been natural to include a discussion about methods for solving nonlinear systems of algebraic equations. Other topics which are treated very briefly or not at all, are explicit difference schemes, higher order schemes and the choice of boundary conditions for approximations where this is not trivial.

The book is written in a very clear and nice way. It is well suited to give a good understanding of the methods treated, and it should be a good help to physicists and engineers who already have some knowledge of practical problems.

BERTIL GUSTAFSSON

Department of Computer Science
Uppsala University
Uppsala, Sweden

2 [6.15, 12.05.1].—KENDALL E. ATKINSON, *A Survey of Numerical Methods for the Solution of Fredholm Integral Equations of the Second Kind*, Society for Industrial and Applied Mathematics, Philadelphia, 1976, vii + 230 pp., 25.5cm. Price $17.50.

This volume surveys some of the numerical methods available for solving, mainly, the equation

$$\lambda x(s) - \int_a^b K(s, t)x(t)\,dt = y(s), \qquad -\infty < a \leqslant s \leqslant b < +\infty.$$

Emphasis is placed on methods which allow rather general kernels K. The survey includes mathematically rigorous error analyses which are done with an eye toward their usefulness for a priori estimation. Computational aspects are also treated, and a number of illustrative numerical examples given, In the end, flowcharts and FORTRAN listings for implementations of two methods are reproduced.

I shall now briefly describe the contents.

The first thirty pages, Part I, are devoted to a review of basic results from functional analysis, necessary for the mathematical development.

In Part II, the first two chapters treat a host of different methods: Successive approximation, Degenerate kernel methods–including ways of obtaining the degenerate kernels, Projection methods–the collocation and Galerkin methods.

In Chapter 3 the author considers what he calles the Nyström method: Assume that for $n = 1, 2, \ldots$ we are given points $t_{j,n} \in [a, b]$, $j = 1, \ldots, n$, and an approximate integration procedure

$$\int_a^b f(s)\,ds \sim \sum_{j=1}^{n} w_{j,n} f(t_{j,n}). \tag{1}$$

Then obtain $z_{i,n}$, which should approximate $x(t_{i,n})$, by solving the system of equations

$$\lambda z_{i,n} - \sum_{j=1}^{n} w_{j,n} K(t_{i,n}, t_{j,n}) z_{j,n} = y(t_{i,n}), \qquad i = 1, \ldots, n.$$

This method is analyzed for continuous kernels K, using a crucial observation of Nyström's. Extensions to the case of singular kernels are given (the product integration method). In the analysis, the notion of collectively compact operators is used, following Anselone and Moore.

The author then notes, in Chapter 4, that the Nyström method probably leads to larger linear systems of equations than the degenerate kernel or projection methods, equal accuracy being demanded. These systems are not sparse, but the author feels that the Nyström method is competitive if certain iterative procedures for solving the linear systems are employed. These procedures are analyzed.

Finally, in Chapter 5, the author discusses computer programs implementing the Nyström method, combined with an iteration procedure given by Brakhage. Two programs are given, the first with the numerical integration scheme (1) being Simpson's rule, the second with Gaussian quadrature.

The author succeeds in treating, and making interplay between, the mathematical and the computational aspects of the surveyed methods, while maintaining high mathematical rigor and giving useful numerical considerations. His value judgements, e.g., his